Full Kinetic Analysis of Small-scale Magneto Plasma Sail in Magnetized Solar Wind

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Abstract: Magneto Plasma Sail (MPS) is spacecraft propulsion that produces an artificial magnetosphere to block solar wind particles, and thus imparts momentum to accelerate a spacecraft. In the present study, we conducted three-dimensional particle-in-cell simulations on small-scale magnetospheres to investigate thrust characteristics of MPS, in which the magnetosphere is inflated by an additional plasma injection. As a result, we revealed that finite thrust generation and the increase in thrust is obtained in the small-scale magnetosphere even if the electron kinetics is taken into consideration. The thrust of MPS (0.58 mN @ magnetic moment M=1.3×10⁷ Wb·m) becomes up to 97 times larger than that of the original magnetic sail (6.0 µN). It was also revealed that the thrust gain of MPS (thrust of MPS / (thrust of magnetic sail + thrust of plasma jet)) is more than unity (up to 5.2). However, the relation of trade-off between specific impulse, thrust-mass ratio and thrust-power ratio is revealed and the optimal design of the spacecraft and missions are required for the realization of MPS.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>A</td>
<td>vector potential</td>
</tr>
<tr>
<td>B</td>
<td>magnetic field</td>
</tr>
<tr>
<td>c</td>
<td>light speed</td>
</tr>
<tr>
<td>dt</td>
<td>time step</td>
</tr>
<tr>
<td>dx</td>
<td>grid spacing</td>
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**F**\text{mag} = \text{thrust of magnetic sail}

**F**\text{MPS} = \text{thrust of MPS}

**λ**\text{D} = \text{Debye length}

**L**\text{MHD} = \text{magnetosphere size}

**µ**0 = \text{magnetic permeability}

**q** = \text{charge}

### 1. Introduction

MAGNETIC sail\textsuperscript{1} and its derivative, Magnetic Plasma Sail (MPS)\textsuperscript{2} are spacecraft propulsion system using the interaction between the magnetic field and the solar wind (Figs. 1 and 2). Using the solar wind, which is supersonic plasma flow mainly consisting of protons and electrons, the sail is expected to be able to provide the efficient thrust in the deep space exploration. Several numerical simulations in order to demonstrate MPS concept, in which a little amount of plasma is injected from the spacecraft in order to inflate magnetic field and obtain larger thrust level, have been performed by MHD and Hybrid Particle-in-Cell (PIC) techniques\textsuperscript{3, 4, 5}. Single fluid approximation is assumed in MHD and electron fluid approximation is assumed in Hybrid-PIC. Hence, the particle kinetics, especially electron kinetics is neglected in these simulation techniques. Therefore, it has not been revealed yet that whether there is any thrust gain by using MPS with a small magnetosphere (<100 km) where electron kinetics cannot be negligible. In addition, effects of Interplanetary Magnetic Field (IMF) have been also ignored in previous studies of MPS. Fully kinetic simulation, such as Full-PIC simulation, which treats both and electron as particle and can solve the plasma flow around the such a small magnetosphere self-consistently, is required to reveal physical phenomena about MPS and to confirm the increase in thrust\textsuperscript{6}. We have already revealed the thrust characteristics of two-dimensional MPS\textsuperscript{7} and three-dimensional magnetic sail (without plasma injection)\textsuperscript{8}.

In this study, we performed large three-dimensional Full-PIC simulations, which have only recently become possible owing to the improvement on the parallel computing techniques, in order to evaluate the thrust force generated by a MPS spacecraft including IMF\textsuperscript{9}.

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**Figure 1.** Schematic illustration of Magnetic Sail and MPS. MPS inflates the magnetosphere around spacecraft by a little plasma injection.

**Figure 2.** Demonstrator spacecraft of MPS proposed by JAXA MPS working group. The superconductive coil (r\text{coil} = 2 m, I\text{coil} = 200 A x 1000 Turn) is shaded from solar radiation.
II. Numerical Model

A. Assumptions

The size of the artificial magnetosphere can be derived from the pressure equilibrium between the magnetic pressure of the dipole magnetic field and the solar wind dynamic pressure as

\[ L_{\text{MHD}} = \left( \frac{\mu_0 M^2}{16\pi^2 m_i N_{\text{SW}} v_{\text{SW}}^2} \right)^{1/6} \]  

based on the MHD approximation. When the typical solar wind parameters listed in Table 1 are assumed, the ion Larmor radius and electron Larmor radius at the magnetopause (magnetosphere boundary) are calculated as \( r_{iL} \sim 100 \) km and \( r_{eL} \sim 50 \) m, respectively. If the magnetosphere size and the ion Larmor radius satisfy the condition, \( L_{\text{MHD}} > r_{iL} \), the solar wind flow can be treated as a single fluid on the MHD scale (Fig. 3). If the magnetosphere size is comparable with the ion Larmor radius, \( L_{\text{MHD}} \sim r_{iL} \), the ion kinetics should be considered and the electron fluid approximation is valid on the ion inertial scale. When the magnetosphere size is smaller than the ion Larmor radius, \( L_{\text{MHD}} < r_{iL} \), both ion kinetics and electron kinetics should be taken into the consideration. In addition, the magnetosphere size becomes comparable with the Debye length (\( \lambda_D \sim 10 \) m) and the charge separation between ions and electrons should be considered on the electron inertial scale.

<table>
<thead>
<tr>
<th>Solar wind velocity ( v_{\text{sw}} ) [m/s]</th>
<th>Solar wind density ( N_{\text{sw}} ) [m(^{-3})]</th>
<th>Plasma temperature ( T_{\text{sw}} ) [eV]</th>
<th>Interplanetary magnetic field [nT]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 5 \times 10^4 )</td>
<td>( 5 \times 10^4 )</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

Figure 3. Simulation scale of the artificial magnetosphere and schematic illustration of trajectories of ion and electron around magnetopause.

Table 1 Typical parameters of the solar wind
B. Full-PIC model

Full-PIC simulation treats both ions and electrons as particles in order to consider the kinetic effects of plasma particles such as finite Larmor radius and charge separation. Full-PIC simulation solves the equation of motion,

$$m_s \frac{d\mathbf{v}_s}{dt} = q_s \left( \mathbf{E} + \mathbf{v}_s \times \mathbf{B} \right) \quad s = \text{ion, electron}$$  \hspace{1cm} (2)

and traces the precise motion of each particle using the Buneman-Boris method. In this study, only two kinds of particles, namely, ions (protons) and electrons, are used and the mass ratio is set precisely \(m_i/m_e = 1836\). From the particle trajectories, the density distribution and current distribution are calculated by the PIC weighting method. Maxwell’s equations:

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$  \hspace{1cm} (3)

and

$$\frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{B} - \mu_0 \mathbf{J}$$  \hspace{1cm} (4)

are solved by the finite-difference time-domain method to obtain a self-consistent electromagnetic field. Computational load becomes huge in order to calculate the trajectory of huge numbers of plasma particles. That has prevented from simulating the plasma flow around small-scale magnetic sail and obtaining the thrust characteristics.

The interaction between the solar wind and an artificial dipole magnetic field is simulated in three-dimensional space. Figure 4 shows the computational domain used in the three-dimensional Full-PIC simulations. The computational domain has an area of 3.8 km \(\times\) 3.8 km \(\times\) 3.8 km partitioned into a grid of 256 \(\times\) 256 \(\times\) 256 cells \((dx = 15\text{ m})\) in the typical case. The grid spacing \(dx\) typically needs to be set such that it is not much larger than the Debye length \((dx/\lambda_D < 3)\). The solar wind plasma is typically represented by 16 super particles associated with each cell, since 16 particles are necessary and sufficient in terms of the calculation cost and correctness of the simulation. The magnetic field generated by the coil mounted in the spacecraft is approximated by an ideal dipole magnetic field of magnetic moment \(M\). The magnetic moment of the coil is arranged in the \(z\)-direction (parallel to the solar wind).

Absorbing boundary conditions are used for the electromagnetic field on all boundaries. Solar wind particles flow into the computational domain from the inflow boundary at the typical solar wind velocity \(V_{SW}\) and thermal distribution. Outgoing particles from the computational domain are eliminated from the calculation.

C. Computational techniques

Full-PIC model requires large computational resource in order to trace large number of particles. In the typical case, 40 billion particles are included in the computational domain. Our simulation code is parallelized by using both OpenMP and Message Passing Interface (MPI). As shown in Fig. 4a, the computational domain is divided to small sub region along \(z\)-axis. Each sub region is calculated by one process. The electromagnetic field and particles stranding the boundary is communicated by MPI. The particle involved in the sub region is divided to smaller groups as shown in Fig. 4b. Thus, two kinds of parallelization techniques are combined. In addition, the load balancing, the particle sorting and the optimization of pipelining are performed.

It takes about four days by using 1024 CPUs to obtain the steady state of the plasma flow.
III. Simulation Result

A. Typical simulation of Magneto Plasma Sail

In order to improve the thrust generation using the solar wind, Winglee et al. proposed M2P2 based on the frozen-in of plasma to the magnetic field (frozen-in concept, high β plasma injection). However, the concept is not valid at all in the small-scale magnetosphere since electron kinetics prevents plasma jet from moving along the magnetic field. Instead of the frozen-in concept of Winglee, the use of dipole plasma equilibrium (equatorial ring-current concept\textsuperscript{10}, low β plasma injection) is proposed. It is expected that the plasma injected from a MPS spacecraft remains in the equatorial plane and induces the diamagnetic current by several drift motions of plasma (Fig. 5). The diamagnetic current flows into the same direction with the coil current to enhance the original magnetic field generated by coil current. We have already demonstrated the ring current concept by two-dimensional simulation\textsuperscript{7} and in order to obtain the thrust characteristics of MPS, we start three-dimensional simulations.

Parameters for a three-dimensional MPS simulation to demonstrate the equatorial ring-current concept are listed in Table 2. Figure 6 shows the definition of plasma injection point $R_{jet}$ and injection direction $\theta_{jet}$. The plasma jet is injected on a concentric circle with $R_{jet}$ and the net thrust by the plasma jet is zero. The thrust by plasma jet $F_{jet}$:

$$F_{jet} = \hat{m}_{jet} V_{jet} + \hat{m}_{jet} V_{jet,e}$$

(5)

is hence the virtual thrust corresponding to the thrust when the plasma jet is injected to one way. Parameters for the three-dimensional Full-PIC simulation of a MPS are restricted of a computational resource. High-density plasma injection or a high-energy plasma injection result in very small Debye length or very large Larmor radius and that prevent us from simulating the MPS. The magnetic moment of the on-board coil is set parallel to the solar wind and the plasma is injected toward the magnetic equator.
### Table 2 Simulation conditions for Magneto Plasma Sail

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Typical case</th>
<th>Parametric survey</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic moment $M$ [Wb·m]</td>
<td>$1.3 \times 10^8$</td>
<td>$1.3 \times 10^7, 1.3 \times 10^8, 1.3 \times 10^9$</td>
</tr>
<tr>
<td>Gas species</td>
<td>$H_2$</td>
<td>$H_2$</td>
</tr>
<tr>
<td>Radius of injection source $R_{jet}$ [m]</td>
<td>200</td>
<td>50, 75, 125, 200, 275, 350, 400, 460, 600</td>
</tr>
<tr>
<td>Injection direction $\theta_{jet}$ [deg]</td>
<td>-90</td>
<td>-90, 0, 90, 180</td>
</tr>
<tr>
<td>Mass flow rate $\dot{m}$ [kg/s]</td>
<td>$2.7 \times 10^8$</td>
<td>$3.3 \times 10^8, 6.7 \times 10^8, 1.3 \times 10^9, 2.7 \times 10^9, 1.1 \times 10^{10}, 2.1 \times 10^{10}$</td>
</tr>
<tr>
<td>Ion velocity $V_{jet,i}$ [m/s]</td>
<td>$5.0 \times 10^7$</td>
<td>$5.0 \times 10^7$</td>
</tr>
<tr>
<td>Electron velocity $V_{jet,e}$ [m/s]</td>
<td>$1.0 \times 10^8$</td>
<td>$1.0 \times 10^6, 1.0 \times 10^7, 5.0 \times 10^7, 1.0 \times 10^8$</td>
</tr>
<tr>
<td>Temperature $T_{jet}$ [eV]</td>
<td>100</td>
<td>100, 400, 700, 1000, 10000</td>
</tr>
</tbody>
</table>

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**Figure 6.** a) Definition of plasma injection source and b) definition of injection point $R_{jet}$ and injection direction $\theta_{jet}$.

Ion density distributions are shown in Fig. 7. Figures 7a and 7b shows the spatial distribution of ion density by MPS in the steady state when the magnetic moment $M$ is parallel to the solar wind. Figures 7c and 7d shows the spatial distribution of ion density by magnetic sail. Slices across the $xy$-plane are shown. The solar wind flows avoiding the magnetic field and a low-density region forms around the spacecraft at $(x, y, z)=(0, 0, 0)$ despite the loose coupling between the ions and magnetic field because of the large Larmor radius of the ions (~100 km). The magnetosphere is symmetric about the $z$-axis and slightly charge-separated. By the plasma injection of the high mass flow rate, the high injection velocity and low injection temperature, the magnetosphere of MPS inflated from the magnetosphere of magnetic sail since the total amount of plasma in the magnetosphere is increased and the large diamagnetic current is induced. The magnetosphere size of MPS (1160 m) becomes 4.2 times larger than that of magnetic sail (260 m). Here, the magnetosphere size is defined using the peak of ion density. The cross sectional area of “sail” hence becomes 17.6 time larger than that of the original magnetic sail. The thrust of the magnetic sail is calculated as $F_{mag}=0.07 \pm 0.01$ mN from the change in momentum of all particles contained in the computational domain. The thrust of the MPS is calculated as $F_{MPS}=1.0 \pm 0.05$ mN. The thrust becomes approximately 14 times larger than the thrust of the magnetic sail (0.07 mN). Thus, following to the magnetic inflation, the thrust of MPS is increased.
Figure 7. Ion density distribution ($M=1.3\times10^8$ Wb·m, typical case).

Figure 8. One-dimensional slice of a) current density and b) electric field along $y$-axis ($x=0, z=0, M=1.3\times10^8$ Wb·m, typical case).
Figure 8 shows the ion density and current density distributions along $y$-axis ($x=0$, $z=0$). Current is induced where the density gradient is larger. That is, $\nabla p$ drift induces the ring current. The current density of electron (blue line) is larger than the current density of ion (green line) and it was revealed that the carrier of the diamagnetic current is almost electron. Figure 8b represents that large charge separation (difference of ion density and electron density $n_i-n_e$) occurs and the electric field to reduce the charge separation is also observed. In addition to $\nabla p$ drift, $\nabla B$ drift and curvature drift, the $E \times B$ drift motion enhance the diamagnetic current and large increase in thrust is obtained.

Next, we introduced the IMF into the MPS simulations to confirm that the IMF can be neglected in MPS. The simulation parameters are same with Table 2 (typical case). The northward IMF and the southward IMF are added. The magnetic flux density is set as 100 nT, which is 20 times larger than the typical value of IMF magnitude. The results are shown in Fig. 9. From Fig. 10, the magnetosphere size of Magneto Plasma Sail with northward IMF becomes slightly larger than that of no IMF and southward IMF. This is because the pileup magnetic field in northward IMF traps the electron strongly and diffusion is suppressed. The diamagnetic current by the trapped electron makes the magnetosphere larger. As a result, 1.1 mN (northward IMF) and 0.92 mN (southward IMF) is obtained. The thrust of Magneto Plasma Sail without IMF (1.0 mN) is slightly changed by the strong IMF. However, as long as we consider the typical magnitude of IMF, the effects of IMF on the MPS can be neglected.

Figure 9. Magneto Plasma Sail a) without IMF, b) with southward IMF and c) northward IMF ($M=1.3 \times 10^8 \text{ Wb} \cdot \text{m}$, typical case).

Figure 10. One-dimensional ion density distribution along $y$-axis ($z=0$, $M=1.3 \times 10^8 \text{ Wb} \cdot \text{m}$, typical case).
B. Parametric study

We performed MPS simulations with various magnetic moment and various plasma injection parameters as listed in Table 2. No only $M=1.3 \times 10^8$ Wb-m we performed above section, but also various magnetic moments are assumed. These are corresponding to $3.3 \times 10^{-3} < L_{\text{MHD}}/r_{\text{IL}} < 3.3 \times 10^{-2}$. Here, $r_{\text{IL}} \sim 100$ km is defined at the magnetopause.

Simulation results about the thrust increase and the thrust gain are shown in Fig. 11. As the magnetic moment, that is, magnetosphere size, becomes larger, the increase in thrust becomes smaller as shown in Fig. 11a. The maximum thrust gain is restricted by the thrust increase. First, the larger thrust gain is obtained by large magnetic moment, and when the magnetic moment is larger, the thrust gain decreases since the thrust increase itself becomes small. In addition, the minimum thrust increase of $M=1.3 \times 10^9$ Wb-m is less than unity. The maximum thrust increase is $F_{\text{MPS}}/F_{\text{mag}} = 96$, that is, the thrust of MPS ($0.58$ mN at $M=1.3 \times 10^7$ Wb-m) is 96 times larger than thrust of magnetic sail in the optical case ($6.0$ µN). However, the thrust gain is only $F_{\text{MPS}}/(F_{\text{mag}}+F_{\text{jet}}) = 0.3$ and thrust generation efficiency is very low. On the other hand, the maximum thrust gain is obtained at $M=1.3 \times 10^9$ Wb-m as $F_{\text{MPS}}/(F_{\text{mag}}+F_{\text{jet}}) = 5.2$ as shown in Fig. 11b. Using the magnetic inflation, MPS can achieve the larger momentum transfer from the solar wind than magnetic sail and large thrust is obtained.

![Figure 11. a) Thrust increase and b) thrust gain with various magnetic moments $M$.](image-url)
IV. Discussion

A. Derivation of characteristic parameters

First, one-dimensional theoretical approach is also performed to distinguish the characteristic parameters of magnetic inflation\textsuperscript{11}. We assumed $m=m_i=m_e$ for simplicity. The boundary condition for plasma injection is described as $v_0, n_0, B_0$ and $A_0$ in $x<0$ as shown in Fig. 12. Variables $n, v_x, v_y, B$ and $A$ are function of $x$.

The energy conservation, the generalized momentum conservation and the equation of continuity are represented as

\[ v_x^2 + v_y^2 = v_0^2 \]
\[ mv_y + qA = qA_0 \]
\[ nv_y = n_0v_0 \]

respectively. The magnetic field is represented as

\[ \frac{dB}{dx} = -\frac{\mu_0 nqv_y}{\sqrt{v_0^2 - v_y^2}} \]

using the plasma density and velocity. The derivative of magnetic field is also described as

\[ \frac{dB}{dx} = \frac{dB}{dA} \frac{dA}{dx} \frac{dB}{dA} = \frac{d}{dA} \left( \frac{B^2}{2} \right) \]

using vector potential. By substituting $v_y$ (Eq. (7)) into Eq. (10),

\[ \frac{d}{dA} \left( \frac{B^2}{2} \right) = \frac{n_0 q^2}{m} \left( A - A_0 \right) \sqrt{1 - \left( \frac{q}{mv_0} (A - A_0) \right)^2} \]

is obtained. By integrating Eq. (11) on the proper boundary condition,
\[
\frac{B^2}{2} = n_0 m v_0^2 \left\{ 1 - \sqrt{1 - \frac{q(\Lambda - A_0)}{mv_0}} \right\} + \frac{B_0^2}{2}
\]  
(12)

is derived. Here, we introduce two variables:

\[
B_{jet} = \sqrt{2\mu_0 n_0 m v_0^2}
\]
(13)

and

\[
\beta_{jet} = \left( \frac{B_{jet}}{B_0} \right)^2
\]
(14)

. We can obtain

\[
\frac{dA}{dx} = B = B_{jet} \sqrt{1 + \frac{1}{\beta_{jet}} - 1 \left[ \frac{q(\Lambda - A_0)}{mv_0} \right]^2}
\]
(15)

from Eq. (12). We also introduce three variables:

\[
\gamma_{jet} = \frac{qA_0}{mv_0}
\]
(16)

, 

\[
s = \frac{qA}{mv_0} - \gamma_{jet} - \gamma_{jet} < s < 1
\]
(17)

and

\[
r_{jet} = \frac{mv_0}{qB_{jet}}
\]
(18)

. Finally, the distribution of plasma and magnetic field along x-axis can be obtained by solving

\[
\frac{dx}{r_{jet}} = \frac{ds}{r_{jet} \sqrt{1 + 1 - s^2}}
\]
(19)

. Although Eq. (19) cannot be solved analytically and the numerical technique is required, it can imagine easily that the solution is dependent on \(B_{jet}, r_{jet}, \beta_{jet}, \gamma_{jet} \) and \(\beta_{jet} \) and \(\gamma_{jet} \) is non-dimensional quantity. These variables do not change by one dimension, two dimensions, or three dimensions, either. As a result, the characteristic parameters of MPS are revealed via the above theoretical analysis.

We hence coordinate all simulation cases by \(B_{jet}, r_{jet}, \beta_{jet} \) and \(\gamma_{jet} \) which are derived from one-dimensional theoretical analysis of plasma inflation. Results are shown in Fig. 13. The horizontal axis represents \(r_{jet} \) and the contour level represents the thrust gain. The double logarithmic plot is used. As a result, it is found that the large thrust gain is obtained around \((B_{jet}, r_{jet}, \beta_{jet}, \gamma_{jet}) = (10^{-7} \text{ T}, 3000 \text{ m}, 0.01, 1)\). In the design of MPS, such conditions of plasma injection are ideal. The result that plasma injection of low \(\beta_{jet} \) is effective is in agreement with the result of two-dimensional analysis7).
B. Propulsive characteristics of Magneto Plasma Sail

We also examined the propulsive characteristics of MPS: specific impulse $I_{sp}$, thrust-mass ratio and thrust-power ratio. Figure 14 represents the specific impulse and the thrust-power ratio of three-dimensional MPS by the constant magnetic moment $M = 1.3 \times 10^8 \text{ Wb} \cdot \text{m}$. The high specific impulse is obtained since the MPS converts the solar wind momentum to the thrust. In contrast, the thrust-power ratio is low. The thrust-power ratio mainly depends on the injection velocity $V_{jet,e}$ since the electric power required to accelerate plasma jet:

$$E_p = \frac{1}{2}(m_{ei}V_{jet,e}^2 + m_{e}V_{jet,e}^2)$$

becomes larger as the injection velocity becomes higher. Thrust efficiency defined as

$$\eta_t = \frac{F}{2mE_p}$$

is very low in almost all cases since the ion that account for most of the mass flow rate does not work to inflate the magnetosphere. The line representing $\eta_t = 100\%$ is shown in Fig. 14.

The thrust-mass ratio of MPS is also calculated based on the present superconductive technology. The weight of the superconductive coil with $M = 1.3 \times 10^7 \text{ Wb} \cdot \text{m}$, $R_{coil} = 2 \text{ m}$ is calculated as 200 kg $^{[12,13]}$. The thrust-mass ratio of three-dimensional MPS (0.58 mN / 200 kg = $2.9 \times 10^{-3}$ mN/kg) is further smaller than that of other existing propulsion system: 0.07 mN/kg of IKAROS (solar sail, only reflecting mirror), 0.40 mN/kg of HAYABUSA and 1.9 mN/kg of Deep Space 1 (ion engine, only thrust system without fuel). However, by the improvement of the superconductive technology and the optimal design of the superconductive coil to magnetic sail $^{[12,13]}$, it is expected that the thrust-mass ratio should be improved by 2~5 times compared with the assumption of this study. By adopting expansion coil structure, the thrust-mass ratio should be improved up to 0.15 mN/kg with $R_{coil} = 100 \text{ m}$.

Figure 15a shows the thrust gain of three-dimensional MPS with various plasma injection parameters (Table 2) summarized by the specific impulse and the thrust-power ratio. The double logarithmic plot is used. The propulsion efficiency becomes high when both the specific impulse and the thrust-power ratio are high. The high thrust gain is obtained when the thrust efficiency is high. Therefore, the effective thrust of MPS is simultaneously obtained by realizing high propulsion efficiency. On the contrary, the thrust-mass ratio becomes high when the specific impulse and the thrust-power ratio are low as shown in Fig. 15b. Therefore, the high acceleration and high thrust efficiency cannot be achieved at same time. This is mainly because we chose the high electron velocity as the plasma injection parameters. Instead of the high electron velocity, the high plasma injection density also can achieve the best plasma injection parameters ($B_{jet}, r_{jet}, \beta_{jet}, \gamma_{jet}$) = ($10^{-7}$ T, 3000 m, 0.01, 1). However, the high plasma injection causes the very short Debye length and it is completely insufficient in the computational resource of the existing super computers.

![Figure 13. Thrust gain summarized by characteristics parameters of magnetic inflation.](image-url)
V. Conclusion

We performed three-dimensional Full-PIC simulations with and without Interplanetary Magnetic Field in order to determine the thrust characteristics of small-scale Magneto Plasma Sails. We provided the one-dimensional theoretical analysis results about magnetic inflation and the four characteristic parameters: Larmor radius $r_{jet}$, plasma injection energy $B_{jet}$, kinetic beta $\beta_{jet}$, and generalized momentum $\gamma_{jet}$ were distinguished. The three-dimensional simulations with various plasma injection parameters and Magneto Plasma Sail design parameter (magnetic moment) were also performed to reveal the propulsive characteristics. The maximum increase (thrust of MPS / thrust of magnetic sail) is 97 but the thrust gain (thrust of MPS / (thrust of magnetic sail + thrust of plasma jet)) is only 0.4 by same condition. On the contrary, it was revealed the maximum thrust gain 5.2 is obtained around $(B_{jet}, r_{jet}, \beta_{jet}, \gamma_{jet}) = (10^{-7} \, T, 3000 \, m, 0.01, 1)$. The thrust characteristics of Magneto Plasma Sail such as the thrust-mass ratio, the specific impulse and the thrust-power ratio are also examined by using the thrust characteristics obtained from the Full-PIC simulations. The relation of trade-off between the thrust-mass ratio, the specific impulse and the thrust-power ratio was revealed.

For the future works, the optimal design of the spacecraft and missions are required for the realization of Magneto Plasma Sail. The improvement of the low thrust-power ratio is especially required to make Magneto Plasma Sail the attractive thrust system for the deep space exploration.
Acknowledgments

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References